

**ARRANGEMENTS IN TICINO 2022**  
**ABSTRACTS OF TALKS AND POSTERS**

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## PART I – TITLES AND ABSTRACTS OF TALKS

**Takuro Abe (Kyushu).**

*Structure of logarithmic derivation and differential modules close to free*

*Abstract.* TBA

**Daniel C. Cohen (Louisiana State).***An introduction to topological complexity of motion planning*

*Abstract.* Motivated by the motion planning problem from robotics, topological complexity is a mathematical invariant of the space of all configurations of a mechanical system. This invariant provides a measure of the complexity of designing a motion planning algorithm for the mechanical system, that is, the complexity of navigation in the configuration space. I will introduce this notion (as well as a recent generalization if time permits), and provide illustrations of its determination. Examples of configuration spaces considered will include familiar topological spaces such as spheres and surfaces, and possibly some spaces of interest in the theory of hyperplane arrangements.

**Patricia Commins (Minnesota).***Invariant Theory of Left-Regular Band Algebras*

*Abstract.* Given a group  $G$  acting on an algebra  $A$ , two classical questions from invariant theory are (1) what is the structure of the invariant subalgebra  $A^G$ , and (2) what does  $A$  look like simultaneously as an  $A^G$ -module and a  $G$ -representation? Setting  $G$  to be a real reflection group and  $A$  to be the face semigroup algebra of the corresponding hyperplane arrangement, question (1) was answered by Bidigare, who proved that the invariant subalgebra is (anti-)isomorphic to Solomon's descent algebra. Question (2) is a work-in-progress and has so far seen only partial results.

Face semigroups are a special example of a larger class of semigroups called "left-regular bands." Motivated by understanding question (2) better for reflection arrangements, we answer both questions for two left-regular bands where the story is simpler: the free left-regular band with action of the symmetric group, and a  $q$ -analogue with the action of a finite general linear group. The story in these cases makes use of random walks on semigroups, and in particular random-to-top shuffling and a  $q$ -analogue. This is joint work with Sarah Brauner and Vic Reiner.

**Michael Cuntz (Hannover).***On connected Subgraph arrangements**Abstract.* TBA**Alessio D'Alì (Osnabrück).***An introduction to symmetric edge polytopes*

*Abstract.* A symmetric edge polytope is a lattice polytope obtained from the combinatorial data of a given finite simple graph. Such a class has seen a lot of interest in the last few years both for its intrinsic combinatorial and geometric properties and for its connections to finite metric space theory, optimal transport and physics. From a combinatorial perspective, symmetric edge polytopes enjoy many pleasant properties: for instance they are reflexive, admit a regular unimodular triangulation and have a unimodal  $h^*$ -vector. However, many questions remain unsolved: for instance, it has been conjectured by H. Ohsugi and A. Tsuchiya that a stronger condition than  $h^*$ -unimodality holds, namely, that the gamma-vector of any symmetric edge polytope always has nonnegative coefficients. The aim of this talk is to provide a gentle introduction to symmetric edge polytopes, and to highlight some open problems in the process. If time permits, we will mention a generalization of the symmetric edge polytope setting to regular matroids. This is joint (and, in some cases, ongoing) work with E. Delucchi, M. Juhnke-Kubitzke, M. Koch, D. Köhne, M. Michałek and L. Venturello.

**Graham Denham (Western Ontario).***Geometry of log derivations on arrangements*

*Abstract.* TBA

**Galen Dorpalen-Barry (Bochum).**

*TBA*

*Abstract.* The coefficients of the Poincaré polynomial of an arrangement in a real vector space have many interpretations. An interesting one is provided by the Varchenko-Gel'fand ring, which is the ring of functions from the chambers of the arrangement to the integers with pointwise multiplication. Varchenko and Gel'fand gave a simple presentation for this ring, along with a filtration whose associated graded ring has its Hilbert function given by the coefficients of the Poincaré polynomial.

We generalize these results to cones defined by intersections of halfspaces of some of the hyperplanes and consider an interesting special case: Weyl cones of Shi arrangements. We give a simpler basis for the Varchenko-Gel'fand ring of a Weyl cone, and find that the coefficients of the cone Poincaré polynomial of a Weyl cone are interpreted by antichains in the root poset. This is joint work with Christian Stump.

**Clément Dupont (Montpellier).**

*Geometry of algebraic Mellin transforms.*

*Abstract.* I will speak of work in progress (with F. Brown, J. Fresan, M. Tapuskovic) on the geometric structures which underlie algebraic Mellin transforms. They are formal versions of the cohomology groups of rank one local systems induced by invertible functions (on complements of hyperplane arrangements, for instance). Our results shed light on a (motivic) Galois theory for the expansions of algebraic Mellin transforms, which is governed by algebraic groups with coefficients in fields of Laurent series.

**Christopher Eur (Harvard).**

*Tautological classes of matroids*

*Abstract.* We introduce certain classes on permutohedral varieties which we call "tautological classes of matroids" as a new geometric framework for studying matroids. Using this framework, we unify and extend many recent developments in matroid theory arising from its interaction with algebraic geometry. We achieve this by establishing a Chow-theoretic description and a log-concavity property for a 4-variable transformation of the Tutte polynomial, and by establishing an exceptional Hirzebruch-Riemann-Roch-type formula for permutohedral varieties that translates between K-theory and Chow theory. Joint work with Andrew Berget, Hunter Spink, and Dennis Tseng.

**Michael Falk (Northern Arizona University).**

*Hypergraphs and Orlik-Solomon algebras*

*Abstract.* We define the Orlik-Solomon algebra of a hypergraph and establish an elementary decomposition theorem. This is sufficient to allow a classification of Orlik-Solomon algebras of matroids using the hypergraphic generalization of Whitney's 2-isomorphism theorem for graphs, due to Vertigan and Whittle (1997). The approach gives a new perspective on local and non-local resonance and leads to direct connections with exterior Stanley-Reisner rings and small covers.

**Nir Gadish (Michigan).**

*Applications of polynomiality in compact support cohomology of configurations on graphs.*

*Abstract.* The compactly supported cohomology of configurations on graphs contributes to the cohomology of moduli spaces of algebraic and tropical curves with marked points. The relevant structure in this calculation is an action of the group of outer automorphisms of a free group on

the cohomology. I will discuss joint work with Hainaut, showing that these representations are essentially polynomial functors in a sense I will explain, and how these functors determine the cohomology of moduli spaces. As a consequence, for genus 2 curves we find huge unexpected though familiar representations of the symmetric group, in both even and odd dimensional cohomology.

**Giovanni Gaiffi (Pisa).**

*Bases for the cohomology of compactifications of toric arrangements and their combinatorial properties.*

*Abstract.* I will describe how to construct monomial bases for the integer cohomology rings of compact wonderful models of toric arrangements. In the description of the monomials various combinatorial objects come into play: building sets, nested sets, and the fan of a suitable toric variety. I will show some examples computed via a Sage program and then I will focus on the case of the toric arrangements associated with root systems of type A. Here the combinatorial description of these basis offers a geometrical point of view on the relation between some eulerian statistics on the symmetric group. This is a joint work with Oscar Papini and Viola Siconolfi.

**Lukas Kühne (Bielefeld).**

*Matroids and Algebra*

*Abstract.* A matroid is a combinatorial object based on an abstraction of linear independence in vector spaces and underlies any arrangement of hyperplanes. I will discuss how matroid theory interacts with algebra via the so-called von Staudt constructions. These are combinatorial gadgets to encode polynomials in matroids. A main application is concerned with generalized matroid representations as arrangements over division rings, subspace arrangements and probability space representations together with their relation to group theory.

Based on joint work with Rudi Pendavingh and Geva Yashfe.

**Caitlin Lienkaemper (Penn State).**

*Combinatorial Geometry in Neuroscience*

*Abstract.* In this talk, I explore the use of oriented matroids, hyperplane arrangements, and related objects in neuroscience.

We first discuss threshold linear networks (TLNs), dynamic models of neural activity with an underlying hyperplane arrangement. First, we talk about how this hyperplane arrangement determines the fixed points of a TLN, and how we can use matroids and oriented matroids to understand the relationship between a network's graph structure and its fixed points. Next, we talk about how the chambers of this hyperplane arrangement shape the dynamics more broadly.

Next, we discuss combinatorial neural codes, which describe neural activity in terms of which sets of neurons fire together. A combinatorial neural code is convex if it arises as the intersection pattern of an arrangement of convex open sets. Convex neural codes arise in nature from the activity of neurons with convex receptive fields, such as place cells, which track an animal's position in space. Characterizing convex codes is mathematically rich, with connections to discrete geometry in general and oriented matroid theory in particular. We discuss these connections.

**Jon McCammond (UC Santa Barbara).**

*Dual Braids and the Braid Arrangement*

*Abstract.* The braid groups have two well known presentations and two corresponding classifying spaces. The elegant minimal standard presentation is closely related to the classical

classifying space derived from the complex braid arrangement complement (viewed as a complexification of the real braid arrangement). The dual presentation, introduced by Birman, Ko and Lee, leads to a second Garside structure and a second classifying space, but it has been less clear how the dual braid complex is related to the (quotient of the) complexified hyperplane complement, other than abstractly knowing that they are homotopy equivalent. In this talk, I will discuss recent progress on this issue. Following a suggestion by Daan Krammer, Michael Dougherty and I have been able to embed the dual braid complex into the quotient of the complex braid arrangement complement. This leads in turn to a whole host of interesting complexes, combinatorics, and connections to other parts of the field. This is joint work with Michael Dougherty.

**Luca Moci (Bologna).**

*On the cohomology of arrangements of subtori*

*Abstract.* TBA

**Roberto Pagaria (Bologna).**

*The  $S_n$ -action on the Orlik-Terao algebra of type  $A_{n-1}$*

*Abstract.* The symmetric group action on the Orlik-Solomon algebra of type A is well known since the work by Lehrer and Solomon in 1986. The same question about the Orlik-Terao algebra naturally arises. Moseley, Proudfoot and Young (2016) related this problem to the study of the intersection cohomology of a certain hypertoric variety. Moreover, they conjectured that this intersection cohomology is isomorphic as a graded representation (but not as an algebra) to the cohomology of the configuration spaces on  $SU_2$  up to translations. We provide a new virtual description of the Frobenius character for the cohomology of the aforementioned configuration space. We use this formula to prove the Moseley-Proudfoot-Young conjecture. As a consequence we obtain the graded Frobenius character of the Orlik-Terao algebra of type A.

**Giovanni Paolini (Amazon/Caltech).**

*The  $K(\pi, 1)$ -conjecture, part 2*

*Abstract.* TBA

**Gerhard Röhrle (Bochum).**

*Inductive Freeness of Ziegler's Canonical Multiderivations*

*Abstract.* Let  $\mathcal{A}$  be a free hyperplane arrangement. In 1989, Ziegler showed that the restriction  $\mathcal{A}''$  of  $\mathcal{A}$  to any hyperplane endowed with the natural multiplicity  $\kappa$  is then a free multiarrangement  $(\mathcal{A}'', \kappa)$ . The aim of this paper is to prove an analogue of Ziegler's theorem for the stronger notion of inductive freeness: if  $\mathcal{A}$  is inductively free, then so is the multiarrangement  $(\mathcal{A}'', \kappa)$ . This is a report on joint work with T. Hoge.

**Mario Salvetti (Pisa).**

*The  $K(\pi, 1)$ -conjecture, part 1*

*Abstract.* TBA

**Henry Schenck (Auburn).**

*Data meets Topology: an invitation to persistent homology*

*Abstract.* A central problem in science is how to extract meaning from a massive dataset. For example, imagine a dense, dancing cloud of mosquitoes—is there a pattern to their movement? One solution is to freeze the cloud at a moment in time, and then (mathematically) “supersize”

the mosquitoes, fattening each individual until nearby groups coalesce. This is an oversimplification, but captures the core idea of topological data analysis. A fundamental tool in the field is *Persistent Homology* (PH), which allows detection and analysis of underlying structure in large datasets. PH has yielded new insights into areas ranging from visual cortex activity to viral evolution to cancer pathology.

Roughly speaking, the “supersizing” above translates into the construction of a series of nested topological spaces  $X_\epsilon$ , where the diameter of the fattened mosquitoes corresponds to the parameter  $\epsilon$ . The dynamics involved in the changing spaces  $X_\epsilon$  translate into an algebraic structure whose underlying foundation is a polynomial ring in one variable—the fattening parameter  $\epsilon$ . This talk (aimed at a general scientific audience) will give an overview of the algebraic, combinatorial, and topological underpinnings of persistent homology.

**Alex Suciú (Northeastern).**

*Arrangements and lower central series*

*Abstract.* We study the lower central series, the Alexander invariants, and the cohomology jump loci of groups arising as split extensions with trivial monodromy in first homology with appropriate coefficients. We use these techniques to gain further understanding of the Milnor fibration of the complement of a hyperplane arrangement.

**Tan Nhat Tran (Bochum).**

*Arrangements arising from digraphs and freeness of arrangements between Shi and Ish*

*Abstract.* To a given vertex-weighted digraph (directed graph) we associate an arrangement analogous to the notion of Stanley’s  $\psi$ -graphical arrangements and study it from perspectives of combinatorics and freeness. Our arrangement unifies several arrangements in literature including the Catalan arrangement, the Shi arrangement, the Ish arrangement, and especially the arrangements interpolating between Shi and Ish recently introduced by Duarte and Guedes de Oliveira. It was shown that the arrangements between Shi and Ish all share the same characteristic polynomial with all nonnegative integer roots, thus raising the natural question of their freeness. We introduce two operations on the vertex-weighted digraphs and prove that subject to certain conditions on the weight  $\psi$ , the operations preserve the characteristic polynomials and freeness of the associated arrangements. In particular, by applying a sequence of these operations to the Shi arrangement, we affirmatively prove that the arrangements between Shi and Ish all are free, and among them only the Ish arrangement has supersolvable cone. Notably, all of the arrangements between Shi and Ish appear as the members in the operation sequence, thus giving a new insight into how they naturally arise and interpolate between Shi and Ish. This is joint work (arXiv:2108.02518) with T. Abe (Kyushu) and S. Tsujie (Hokkaido).

PART II – TITLES AND ABSTRACTS OF POSTERS

**Ahmed Ashraf (Ghent).**

*Stable intersections and Tutte coefficients*

*Abstract.* Lopéz de Medrano-Rincón-Shaw defined Chern-Schwartz-MacPherson cycles for an arbitrary matroid  $M$  and proved by an inductive argument that the unsigned degrees of these cycles agree with the coefficients of  $T(M;x,0)$ , where  $T(M;x,y)$  is the Tutte polynomial associated to  $M$ . Ardila-Denham-Huh utilized this interpretation of these coefficients to prove their log-concavity. We show that these coefficients can be obtained as the tropical stable intersection numbers of Chern-Schwartz-MacPherson cycles using the point count formula. (This is a joint work with Spencer Backman)

**José Bastidas (Montréal).***The Primitive Eulerian Polynomial*

*Abstract.* We introduce the Primitive Eulerian polynomial  $P_{\mathcal{A}}(z)$  of a central hyperplane Arrangement  $\mathcal{A}$ . It is a reparametrization of the cocharacteristic polynomial of the arrangement. Previous work (2021) implicitly showed that this polynomial has nonnegative coefficients in the simplicial case. If  $\mathcal{A}$  is the arrangement corresponding to a Coxeter group  $W$  of type A or B, then  $P_{\mathcal{A}}(z)$  is the generating function for the (flag)excedance statistic on a particular subset of  $W$ . No interpretation was found for reflection arrangements of type D.

We present an alternative geometric and combinatorial interpretation for the coefficients of  $P_{\mathcal{A}}(z)$  for all simplicial arrangements  $\mathcal{A}$ . For reflection arrangements of types A, B, and D, we find recursive formulas that mirror those for the Eulerian polynomial of the corresponding type. We also present real-rootedness results and conjectures for  $P_{\mathcal{A}}(z)$ . This is joint work with Christophe Hohlweg and Franco Saliola.

**Evienia Bazzocchi (Bologna).***Configuration spaces of graphs and simplicial complexes and their homology*

*Abstract.* Configuration spaces of graphs have been widely studied since early 2000's. In 2001, Świątkowski constructed a cubical complex which is a deformation retract of the unordered configuration space of a finite graph. More recently, using an algebraic version of this model, An, Drummond-Cole and Knudsen found a family of generators for the first homology group of the configuration space of any finite graph and for the second homology group in the planar case. We are trying to solve the same problem for non-planar graphs, using some kind of functoriality given by the minor relation on graphs and not only by the immersion of subgraphs. Passing from graphs to simplicial complexes, we are investigating representation stability phenomena of the homology ordered configuration spaces when the number of particles increases. For closed manifolds, some stability results are obtained by adding new particles near the old ones using the existence of a non-vanishing vector field. Our purpose is to adapt this idea in the simplicial context. This is a joint work with my advisor, Roberto Pagaria.

**Alessio Borzì (Warwick).***Set of independencies and Tutte polynomial of matroids over a domain*

*Abstract.* Associated to any divisor in the Chow ring of a simplicial tropical fan, we construct a family of polytopal complexes, called normal complexes, which we propose as an analogue of the well-studied notion of normal polytopes from the setting of complete fans. We describe certain closed convex polyhedral cones of divisors for which the “volume” of each divisor in the cone—that is, the degree of its top power—is equal to the volume of the associated normal complexes. We view the theory of normal complexes developed in this paper as a polytopal model underlying the combinatorial Hodge theory pioneered by Adiprasito, Huh, and Katz.

**Pragnya Das (Hokkaido).***The generalization of Sylvester's And Orchard Problems Via Discriminantal Arrangement.*

*Abstract.* In 1989 Manin and Schechtman defined the discriminantal arrangement  $B(n, k, A)$  associated to a generic arrangement  $A$  of  $n$  hyperplanes in a  $k$ -dimensional space. In this poster I provide an example of 12 lines with 19 3-points using Pappu's configuration that elucidates the connection between the well known generalized Sylvester's and orchard problems and the combinatorics of  $B(n, k, A)$ . In particular I point out how this connection could be helpful to address those old but still open problems.

**Federica Gavazzi (Bologna).***The  $K(\pi, 1)$ -conjecture and the pure virtual braid group*



*Abstract.* TBA

**Leonie Mühlherr (Bielefeld).**

*Regular subdivisions in log-concave density estimation*

*Abstract.* Cule, Samworth and Steward found that estimating a log-concave density for a given data sample gives rise to a regular subdivision of the polytope spanned by the datapoints. Conversely, it has been shown by Robeva, Sturmfels and Uhler that given a regular subdivision of the convex hull of the data, it is possible to find an open set of weight vectors, such that the regular subdivision arising from the maximum likelihood estimator for the weighed data points is the chosen one. This poster gives an introduction to these results and offers a partial result for the relations between the weight vectors for specific subdivisions as well as an attempt at enhancing test statistics for log-concavity by using the f-vector of the associated subdivision.

**David Munkacsi (Hannover).**

*A tight lower bound for the number of lines in an arrangement with the Mod 2-net property*

*Abstract.* In our masters thesis, we have developed a modified version of an algorithm from Cuntz on a problem originating from Yoshinaga asking if there exist arrangements of lines in the projective plane that we can color satisfying a given property called the Mod 2-net property, i.e. such that we can give a coloring where we can color each projective line with one of two colors so that each intersection point is either the intersection of lines of one color, or the intersection of an even number of lines of each color. We were able to combinatorially determine a tight lower bound for the number of lines in such an arrangement.

**Anastasia Nathanson (Minnesota).**

*Putting Volume back in Volume Polynomial*

*Abstract.* Associated to any divisor in the Chow ring of a simplicial tropical fan, we construct a family of polytopal complexes, called normal complexes, which we propose as an analogue of the well-studied notion of normal polytopes from the setting of complete fans. We describe certain closed convex polyhedral cones of divisors for which the “volume” of each divisor in the cone—that is, the degree of its top power—is equal to the volume of the associated normal complexes. We view the theory of normal complexes developed in this paper as a polytopal model underlying the combinatorial Hodge theory pioneered by Adiprasito, Huh, and Katz.

**Maddalena Pismataro (Bologna).**

*TBA*

*Abstract.* The interest in arrangements and topological properties of their complement finds its roots in the '60, starting with the hyperplane case, followed by the toric and the elliptic ones, up to arriving to their generalization: abelian arrangements. One of the first tools used to investigate the topology is the Salvetti Complex, thanks to which it is even possible to give a presentation of the fundamental group and describe the homotopy type of the complement. Later, respectively in 2007 (Salvetti, Settepanella) and 2021 (d'Antonio, Delucchi), it has been shown that the complement of hyperplane and toric arrangements have the homotopy type of a minimal CW complex, with results also derived from Morse Theory. It should be interesting to study the minimality even in the elliptic case. Lately, a special class of arrangements has been studied, known as “supersolvable” (Delucchi, Bibby, 2022), whose combinatorial properties prove that their complements are  $K(\pi, 1)$ , and led to few open problems to deepen and deal with.

**Takuya Saito (Hokkaido).**

*Classification of discriminantal arrangement  $\mathcal{B}(6, k, \mathcal{A})$  using the symmetric group of degree 6*

*Abstract.* The discriminantal arrangement  $\mathcal{B}(n, k, \mathcal{A})$  is an induced central hyperplane arrangement in space consisting of all translations of a generic arrangement  $\mathcal{A} = \{H_1, \dots, H_n\}$  in  $k$ -dimensional space. The combinatorics of discriminantal arrangement depends on the representation of  $\mathcal{A}$ , not only  $n, k$ . We classified all combinatorics of discriminantal arrangement  $\mathcal{B}(6, k, \mathcal{A})$  by using the symmetric group of degree 6, and we show their type correspond to the integer partition of 6.

**Alessandro Schena (Tromsø).**

*Wachpress conjecture restricted to arrangements of three conics.*

*Abstract.* Wachpress' conjecture states the existence of rational barycentric coordinates for polycons in a real projective space of any dimension. These coordinates are known as Wachpress coordinates. Wachpress coordinates are proven to exist for all planar polytopes and in some other special cases. They offer a way to produce a finite element method of approximation with linear precision. Polycons generalize polytopes and describe objects which are quite common in practical modeling problems. Thus, the existence of these coordinates on a broad set of geometrical objects interests many fields, from engineering to statistics and beyond. In a planar setting, a polycon arises from an arrangement of curves. Its adjoint is the minimal degree curve which intersects the arrangement only in the residual points. To prove the conjecture is to prove that the adjoint of the polycon does not intersect the interior of the polycon. Real polycons can be seen as positive geometries, and the numerator of their canonical form is the adjoint polynomial to the polycon. One can thus use the techniques of positive geometries to aid the task of determining if the adjoint can have a connected component in the interior of the polycon. The first planar unsolved case for the conjecture is for arrangements of three ellipses. In this case the adjoint is a cubic curve. Its real connected components are either a pseudoline or an oval and a pseudoline. There are 44 topologically distinct cases of arrangements of three ellipses. For 33 topological cases the conjecture has been proven thanks to the usage of hyperbolicity for 28 cases and a case distinction for 5 more cases which manifest a special behavior of the adjoint. Our work has been to try another way to produce a topological catalog of arrangements of three conics from a combinatorial point of view. All the results which we refer to have been published in a recent article by Kohn, Ragni, Ranestad and many others. This poster is based on my Master's thesis project, with Cordian Riener as supervisor.

**Avi Steiner (Western Ontario).**

*Geometry of log derivations on arrangements*

*Abstract.* Of interest to people who study both hyperplane arrangements and commutative algebra are the homological properties of the module of logarithmic derivations of a hyperplane arrangement  $A$ . I will present some interesting results and questions about the "ideal of pairs", which is a sort of "symmetrization" of this module of logarithmic derivations with respect to matroid duality. This is an ideal which simultaneously "sees" many of the homological properties of both the arrangement and its dual, and will be discussed in slightly more detail in a talk by Graham Denham.

**Sakumi Sugawara (Hokkaido).**

*Divides with cusps and Kirby diagrams for line arrangements*

*Abstract.* It is known that the complement of complex line arrangement is homotopy equivalent to minimal 2-dimensional CW complex. In this poster, we describe the handle decomposition of the complexified complements of real line arrangements. To describe the Kirby diagrams, we introduce the notion of the divide with cusps which is the generalization of the divide introduced by A'Campo. This is a joint work with Masahiko Yoshinaga.

**Hasan Suluyer (METU).**

*Combinatorics on Heavy  $(3,2n+1)$ -Multinets*

*Abstract.* A  $(k,d)$ -multinet is a certain configuration of lines and points with multiplicities in  $\mathbb{C}P^2$ . If there is at least one multiple line in a class of a  $(k,d)$ -multinet, it is called heavy. By using the main result proved by Yuzvinsky and the article written by Bassa and Kışisel, we conclude that if a multinet is heavy, the only  $k$  value is 3. Therefore, each heavy multinet is of the form  $(3,d)$ . A heavy  $(3,2n)$ -multinet is constructed for  $n \geq 1$ . We discuss the possibilities for combinatorics of lines and points inside a heavy  $(3,2n+1)$ -multinet and have showed that there exists neither a heavy  $(3,3)$  nor a heavy  $(3,5)$ -multinet. Moreover, we have discovered several numerical results of a heavy  $(3,2n+1)$ -multinet containing a multiple line consisting of three points from  $X$ .

**Lorenzo Vecchi (Bologna).**

*Stressed hyperplane relaxation for KL polynomials of matroids*

*Abstract.* Kazhdan-Lusztig polynomials for matroids were defined in 2016 by Elias, Proudfoot and Wakefield in analogy with the ones arising on the Bruhat order of a Coxeter group. Now, together with the Schubert Variety of a matroid, they are a consolidated geometric tool of Matroid Theory that led the way to solve the famous Top-Heavy Conjecture on the lattice of flats of a matroid. These polynomials have a purely combinatorial description, albeit intricate due to a recursive computation on the minors of the matroid; as a consequence, many properties can be observed and conjectured, but are very hard to be proved. My goal has been to define a new operation on matroids, called stressed hyperplane relaxation, that behaved nicely enough with respect to these invariants. This let us give partial positive answers to some conjectures on the predominant classes of paving and sparse paving matroids. All these computations still hold on a higher categorical level, where you can define an equivariant KL polynomial as a graded representation of a finite group acting on the matroid. This is a joint work with L. Ferroni, G.D. Nasr, T. Karn, N. Proudfoot.